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Progress on the Use of Compliant Walls for Laminar-Flow Control

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This paper reviews and assesses the recent progress toward making the use of compliant walls a practical method of laminar-flow control. Three main areas are covered. First, the current understanding of the vitally important flow-induced surface instabilities is assessed. Some new results are included. Then the optimization of multiple-panel compliant walls is considered. New numerical simulation results are included showing that short compliant panels are very effective in suppressing Tollmien–Schlichting waves. It is found that for marine applications appropriately designed multiple-panel compliant walls are capable of suppressing Tollmien–Schlichting waves to indefinitely high Reynolds numbers. Finally, the feasibility of using compliant walls for laminar-flow control in aeronautical applications is assessed. It is found that, although there is no reason in principle why compliant walls cannot be used in air, in practice exceptionally delicate walls are required to obtain the necessary match between the fluid and structural inertias. The resulting lack of robustness for such walls is deemed to make them completely impractical for aeronautical applications.

I. Introduction

THE first research results on the use of compliant walls for laminar-flow control were announced by Kramer[1,2] in 1957 and 1960 in a forerunner of the *AIAA Journal*. A fuller account followed almost immediately in Kramer.[3] Also Gyorgyfalvy's[4] paper, which appeared in the *Journal of Aircraft* in 1967, was the first attempt to consider the practical design of compliant walls for laminar-flow control. Accordingly, it is, perhaps, fitting that an account of the current status of this method of flow control should appear in the *Journal of Aircraft*. In many respects our approach could be considered to follow the path sketched out by Gyorgyfalvy in the light of our much greater contemporary knowledge of the flow physics involved.

Over the years many excellent reviews[5–10] that survey the use of compliant walls from various perspectives have appeared. Accordingly, the present paper will not review the earlier work in depth, but will only include sufficient detail to be self-contained. Our main aim is to review recent progress towards making the use of compliant walls a practical method of laminar-flow control. In many respects it is an update on Ref. 10 and will cover some common ground. However, several new results will be presented. Also we will make an assessment of the practicalities of using compliant walls for aeronautical applications. We will concentrate on three main areas, namely, progress toward understanding the flow-induced instabilities (i.e., the hydro- and aeroelasticity behavior), the optimization of multiple-panel compliant walls for laminar-flow control, and the feasibility of aeronautical applications.

There has been an unfortunate and widespread tendency to use the term *compliant* to describe any flexible wall, coating, panel, etc., even when the wall is actively driven. The term loses its point if it is used as a mere synonym for flexible. In fact, compliance implies that the flow and wall properties are in some way matched. We will use the term to describe a passive flexible wall for which the propagation speed of the free surface waves is the same order of magnitude as the freestream flow speed U_∞. In fact, optimization studies[11] have shown that the best compliant walls for laminar-flow control have free-surface-wave speeds of about $0.7U_\infty$. It will become clear that this is the paramount factor governing the feasibility of aeronautical application. Rather suggestively, the epidermis of the dolphin, Kramer's original inspiration for his compliant coatings, also has surface wave speeds of about 0.7 times its maximum sustained swimming speed.[12,13] We will also briefly discuss some other types of flexible and interactive walls.

For the most part we will focus on laminar-flow control, i.e., the use of compliant walls to postpone or completely suppress transition from laminar to turbulent flow. There are, of course, many different routes to transition involving a bewildering variety of physical mechanisms, depending on environmental and other factors.[14] Here we will confine ourselves to the transition process in the low-disturbance environments encountered for external flows in aeronautical and marine applications. In such cases, for boundary layers which are similar to that over a flat plate, the route to transition begins with the amplification of small-amplitude, quasi-two-dimensional, Tollmien–Schlichting (T/S) waves. In experimental studies such waves are produced artificially as monochromatic wavetrains by a driver, for example, an oscillating ribbon. Likewise most theoretical studies address this artificial situation. It is important, however, to emphasize that T/S waves have been observed many times in natural transition.[15,16] But, as demonstrated originally by Schubauer and Skramstadt,[15] the use of a driver to excite the boundary layer artificially produces much cleaner signals.

In natural transition the small disturbances are created by such environmental factors as freestream turbulence, acoustic radiation, vibration, surface roughness or, particularly in marine applications, through the entrainment of particulate matter into the boundary layer. The processes whereby T/S waves are generated through such sources of natural excitation are known collectively as *receptivity*. Despite their importance, little is known about many of these receptivity mechanisms, especially how they are affected by wall compliance. For the flat-plate boundary layer, after the small-amplitude T/S waves have been created by a receptivity process, they amplify as they propagate downstream and eventually reach a sufficiently large amplitude for nonlinear effects to become significant. At this point the disturbances become three dimensional, and the several stages

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of the transition process proper rapidly ensue. The actual transition zone itself is characterized by turbulent spots, and no direct evidence of the T/S waves remains. Nevertheless, this final stage of transition would not have occured without the initial creation and amplification of T/S waves. In low-disturbance environments the initial amplification of quasi-two-dimensional T/S waves (the so-called linear regime of transition) typically extends over 70–80% of the total transition process. What we aim to achieve with the use of compliant walls for laminar-flow control is to extend this linear regime greatly or even to suppress the growth of T/S waves entirely.

In a series of experimental tests carried out in open water, Kramer[1–3] established that substantial drag reduction (up to 60%) could be achieved with appropriately designed compliant coatings. Although he designed his compliant coatings with the aim of postponing transition, there was no actual evidence that the observed drag reductions were caused by transition delay. The early theoretical work by Benjamin[17,18] and Landahl[19] followed closely on Kramer's first papers. These seminal papers provided the foundation of the subsequent theoretical work. In an important respect Benjamin and Landahl's theories supported Kramer's concept of his compliant coatings acting to postpone transition. They showed that wall compliance can stabilize T/S waves. However, they also showed that wall damping promoted higher growth rates of the T/S waves, suggesting that the lower the wall damping the better from the point of view of laminar-flow control. This idea ran counter to Kramer's concept and his test results. He had incorporated a layer of highly viscous fluid within his coatings in the belief that this would impede the growth of T/S waves. Furthermore his experimental tests showed that there was an optimum choice of damping-fluid viscosity (about 200 times the viscosity of water) for drag reduction.

Benjamin and Landahl made no attempt to model Kramer: compliant coatings. Their theory was general, rather than being aimed at a particular design of compliant wall. Accordingly, a theoretical assessment of the laminar-flow capabilities of Kramer's coatings was lacking. This was supplied by Carpenter and Garrad[20,21] who modeled the Kramer coatings as a plate supported by a spring foundation and also included the effects of viscoelastic damping and of a viscous damping fluid. Their results broadly confirmed that Kramer-type coatings were capable of substantial transition delay. They also showed that wall damping could play a beneficial role, but not on T/S waves as Kramer thought. Although wall damping has a fairly weak adverse effect on T/S waves, it has a much stronger stabilizing effect on one of the additional flow-induced instabilities, which was termed *traveling-wave flutter* in Ref. 21. In effect, increased wall damping permitted a more compliant wall to be used without suffering the onset of traveling-wave flutter.

Substantial theoretical support now existed for the stabilizing effect of wall compliance on T/S waves, but experimental confirmation was lacking. This was provided by Gaster.[22] A schematic sketch of Gaster's compliant panel and experimental setup is given in Fig. 1. The design of the compliant wall was somewhat simpler than that of Kramer; it consisted of two layers: a thin outer, stiff, plate-like layer, backed by a much softer and thicker layer resting on a rigid wall. The experimental model was essentially a flat plate with a compliant-panel insert. The T/S waves were generated by a driver located ahead of the compliant panel and measured at its trailing edge by means of a surface hot-film gauge. The plate was attached to the carriage of a towing tank and propelled through still water over a range of carefully controlled speeds. Only two parameters could be controlled during the experiments, namely, the carriage speed (i.e., flow speed) and the driver frequency. The gain in disturbance amplitude between the driver and the trailing edge of the compliant panel was determined over a range of driver frequencies and flow speeds for three different compliant panels. Close agreement was found between the measured gains and those predicted for the propagation of T/S waves over compliant walls using linear hydrodynamic stability theory. Slightly better agreement was found by Lucey and Carpenter,[23] who incorporated the effects of tensioning the upper plate-like layer in their theoretical model.

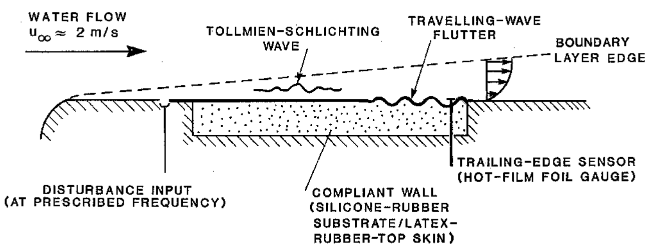

Fig. 1 Setup for Gaster[22] experimental investigation.

An interesting, and highly significant, feature of Gaster's experimental tests was that for the two most compliant of his three panels the route to transition was not amplification of T/S waves. Unlike the relatively gradual process observed for the rigid control, transition occurred suddenly when a critical flow speed was reached. Moreover, when this happened the hot-film gauge located at the panel's trailing edge displayed a signal that oscillated at about three times the driver frequency. It was shown by Lucey and Carpenter[23] that in these cases traveling-wave flutter set in at the observed transition speed. (This essentially wall-based instability is shown schematically in Fig. 1.) Their theoretical model predicted both the onset speeds and frequencies in close agreement with the experimental observations. Thus this established that the theoretical tools, based on linear stability theory, could predict accurately the complex response of compliant walls to both T/S waves and traveling-wave flutter. Furthermore, it was now clear that an understanding of flow-induced instabilities (the significance of which was appreciated and identified in the early theoretical work in Refs. 17–19) is crucial for designing compliant panels for laminar-flow control. Optimization procedures for the design of compliant panels for laminar-flow control were developed by Carpenter and Morris[24] and Dixon et al.[11] The former considered the plate-spring model of Kramer-type walls, and the latter studied the more difficult, two-layer, Gaster-type walls, basing their theoretical approach on the methods developed by Yeo.[25] In both cases in excess of a five-fold rise in transitional Reynolds number was predicted for the best compliant-wall designs. The mechanical and material properties required for operation in water were not greatly different from those of Kramer and were certainly realizable in practice. A better performance could be obtained when an optimal level of wall damping is used to control the traveling-wave flutter. In principle, there is no reason why the theoretical methods could not be used for designing compliant walls for aeronautical applications. However, one would expect the required mechanical and material properties to be quite different for air than for water.

We will consider the question of compliant-wall designs for aeronautical applications as well as more recent developments in the subsequent sections of the paper. The remainder of the paper is set out as follows. Section II describes the recent progress toward a better understanding of the crucially important, flow-induced surface instabilities and their prediction for design purposes. Section III reviews the recent work on the design of multiple-panel compliant walls for laminar-flow control. Section IV considers the feasibility of aeronautical applications of compliant, and other flexible and interactive, walls. Finally, Section V contains brief conclusions and recommendations.

II. Flow-Induced Surface Instabilities

The importance of understanding the flow-induced surface instabilities has already been made clear. As the name suggests, these are essentially instabilities of the compliant wall itself. Some of them are already familiar in the context of hydro- and aeroelasticity. Three main types of such instabilities have been identified, namely, 1) traveling-wave flutter, 2) divergence, and 3) transitional modes.

The first of these is generated by irreversible (i.e., nonconservative) energy transfer to the wall as a result of work done by the fluctuating pressure. The destabilization mechanism, is essentially an inviscid mechanism, and viscous effects are secondary. Divergence takes the form of slowly traveling waves and is also inviscid in origin. The third type comes about owing to a coalescence of a T/S wave and traveling-wave flutter. The term *transitional mode* was coined by Sen and Arora.[26] We will consider each of these three types of instability in turn.

A. Traveling-Wave Flutter

If the boundary layer were ignored and the energy transferred to the compliant wall as a result of work done by the fluctuating pressures in an unsteady potential flow were to be estimated, it would be found that no net work was transfered over a wave period in this conservative system. This is because the vertical wall velocity and pressure are 90 deg out of phase. Plainly if the presence of the boundary layer can alter this phase difference, then there is a possibility of irreversible (i.e., nonconservative) energy transfer to the wall. Benjamin[18] showed that a mechanism originally identified by Miles in connection with water waves could also apply to waves on compliant walls. He showed that when the wave speed was between 0 and U_∞ the required phase change in pressure occurred at the critical point where the local velocity in the boundary layer equals the wave speed. Thus, although viscous effects are required to create the boundary layer and may also play a secondary role, this destabilization mechanism is essentially inviscid. In Ref. 21 it was shown that a knowledge of this mechanism allowed us to provide the following, simple and accurate, estimates for the critical wavenumber and onset flow speed of traveling-wave flutter in the case of the plate-spring model:

$$\alpha_c = (K/B)^{\frac{1}{4}}, \qquad U_c = \left[(2\sqrt{BK} + T)/b\rho_p\right]^{\frac{1}{2}} \tag{1}$$

where K is the spring stiffness, B and T the flexural rigidity and tension per unit width of the plate respectively, and b and ρ_p are respectively the thickness and density of the plate. For walls with no damping, this result holds for all Re_{δ^*} (Reynolds number based on displacement thickness), but when there is damping in the wall it is only valid for infinite Reynolds number. A more rigorous analysis based on asymptotic techniques and including other irreversible mechanisms was developed by Carpenter and Gajjar.[27] This was adapted in Refs. 11 and 23 for the more complex case of Gaster-type two-layer walls. This approach together with Eq. (1) will be used in Sec. IV to assess the feasibility of aeronautical applications.

To summarize the attributes of traveling-wave flutter, it is a convective instability that travels at speeds of the order of $0.7U_\infty$, and it is destabilized by irreversible energy transfer to the wall, whereas energy transfer out of the wall, such as damping, has the opposite effect and can be used to control it. Figure 2 presents the results of a numerical simulation of traveling-wave flutter propagating in the boundary layer over a finite compliant panel of plate-spring type. The figure shows ray paths traced out by a wave packet propagating downstream initiated by creating a small bump on the left of the domain, which is then allowed to relax. It can be seen that, in fact, two separate wave packets are present. Despite appearances, the right-hand one becomes dominant at later times and corresponds more closely to Eq. (1). The computations were carried out using the novel discrete-vortex method described in Ref. 28. Figure 2 is based on Fig. 2 of Ref. 29. It appears that traveling-wave flutter is now well understood and can be confidently predicted with existing theory.

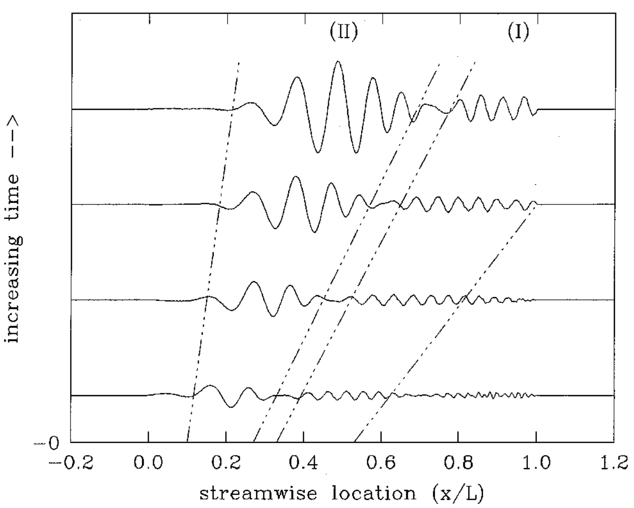

Fig. 2 Wall displacement profiles obtained by numerical simulation of traveling-wave flutter produced by a relaxing bump located near the leading edge of the compliant panel. The flow is from left to right (based on Fig. 2b of Ref. 29).

B. Divergence

In some respects divergence is a simple instability to understand. Imagine a small disturbance in the form of a bump is somehow created on a compliant surface. There will a pressure drop over the bump creating a suction force. If the flow speed is steadily increased, this suction force will rise proportionately to U_∞^2. At sufficiently high flow speed it will outweigh the restorative structural force in the wall, and the bump will grow until checked by the rise in the structural force caused by nonlinear effects. Thus the physical mechanism is conservative and does not require any viscous effects at all; it can occur even in a potential flow. In fact, the following simple estimates for the critical wave number and flow speed were derived in Ref. 21 for potential flow over the plate-spring compliant-wall model:

$$\alpha_d = (K/3B)^{\frac{1}{4}}, \qquad U_d = 2\left(BK\big/27\rho_f^4\right)^{\frac{1}{8}} \tag{2}$$

where ρ_f is the density of the fluid. This appeared to give reasonable agreement with the experimental data then available. According to this model, divergence is a static wave at the point of instability and slowly travels downstream at supercritical flow speeds. It has apparently been observed to occur on dolphins.[30,31]

The phenomenon is not quite so straightforward for the Gaster-type, two-layer walls and for the simpler one-layer walls (Gaster walls with the top plate-like layer removed). For example, Duncan et al.[32] found that for the single-layer walls $\alpha_d = \infty$. Divergence on such walls was fully investigated in a seminal experimental study by Gad-el-Hak et al.[33] They found that the divergence waves traveled slowly downstream at speeds between $0.02U_\infty$ and $0.05U_\infty$. They exhibited sharp crests with broad valleys between each wave. Perhaps the most significant finding was that the divergence waves only occurred when the flow was turbulent. This is most dramatically illustrated in Fig. 10 of Ref. 33 in which a wedge of turbulence surrounded by laminar flow was created by a local roughness element. The divergence waves were only created within the turbulent wedge. This was explained by Duncan et al.[32] by writing the pressure acting on the wall in the form

$$C_p e^{i\theta} p_{\text{pot}} \tag{3}$$

Here p_{pot} is the wall-pressure fluctuation caused by perturbed potential flow, as in the derivation of Eq. (2). $C_p \leq 1$ is the reduction in magnitude of the wall pressure and θ the phase change, both occurring as a result of the presence of the boundary layer. It could then be argued that C_p was much smaller for laminar boundary layers than turbulent ones. However, no method was available to predict C_p. This has now been remedied as will be explained shortly. In a subsequent experimental study using an ultra-low-damping material, Gad-el-Hak[34] was also able to produce traveling-wave flutter on a single-layer wall. In contrast to the preceding case, the waves were much closer to a sinusoid form and travelled at around $0.5U_\infty$.

Figure 3 (taken from Ref. 29) presents a simulation of divergence, which can be compared with Fig. 2. The same technique was used, and, as before, the disturbance was created by a relaxing bump—this time located in the center of the domain. The main difference between Figs. 2 and 3 is that the flow conditions are now such that the flow/wall system is unstable with respect to divergence. It can be seen that the ray patterns and displacements are quite different from Fig. 2. Now the wave packet does not propagate downstream but spreads in both directions from the point of initiation. This is characteristic of an absolute instability. In fact, Yeo et al.[35] have

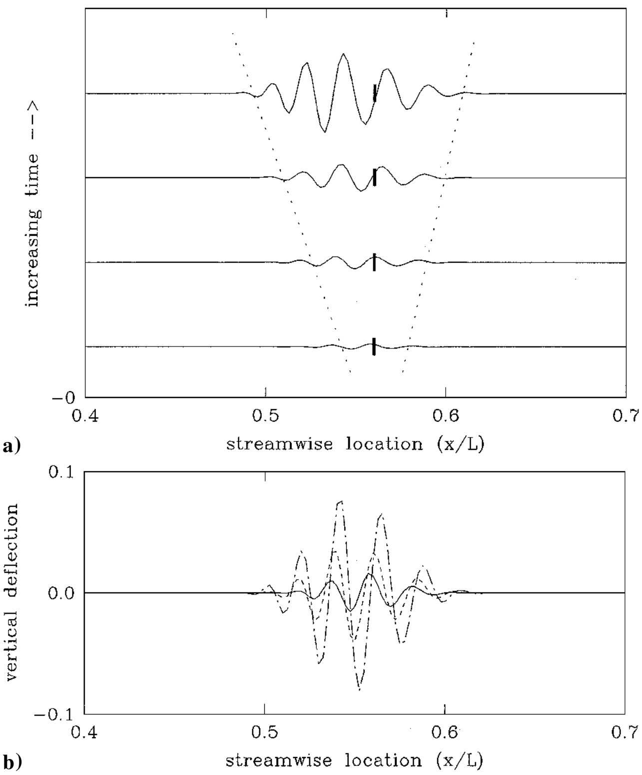

Fig. 3 Numerical simulation of divergence: a) space-time form; b) wall displacement profiles at successive times (based on Fig. 4 of Ref. 29).

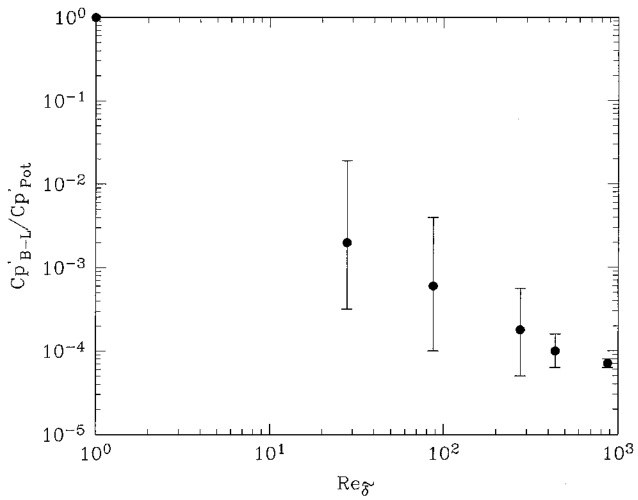

Fig. 4 Pressure scaling coefficient plotted against Reynolds number based on shear thickness for a wall disturbance in the form of a static short wave.

formally shown that divergence is an absolute instablity. By means of these simulations, we have been able to determine the coefficient C_p as a function of $Re_{\tilde{\delta}}$ (Reynolds number based on shear thickness $\tilde{\delta}$) for a static wave of length λ, keeping U_∞ and the velocity-profile shape fixed. (The shear thickness $\tilde{\delta} = U_\infty \mu/\tau_w$, where μ is dynamic viscosity and τ_w is the shear stress at the wall.) An example of the variation of C_p vs $Re_{\tilde{\delta}}$ is plotted in Fig. 4 for a short wall wave. Turbulent boundary layers have a small shear thickness and, accordingly, relatively high values of C_p, whereas laminar boundary layers have large values of shear thickness and relatively small values of C_p. The trends shown in Fig. 4 are exaggerated; in practice we would expect $C_p \geq 0.1$.

The peculiar shape of the divergence waves revealed in the experiments of Gad-el-Hak et al.[33] has recently been explained by Lucey et al.[36] They showed that it is necessary to include nonlinear effects in both the fluid and wall dynamics. When this was done, the hydrodynamic stiffness became increasingly peaky, similar to the waveforms observed in the experiments, as the wave amplitude increased.

C. Transitional Mode

Under certain circumstances T/S waves can coalesce with the traveling-wave flutter instability to form a much more powerful instability,[8] which was termed a *transitional mode* by Sen and Arora.[26] It appears[20] that excessive use of wall damping to control traveling-wave flutter can give rise to this instability. Thus it sets an upper limit on the level of damping that can be used to control traveling-wave flutter. But wall damping is not essential for its existence. A numerical simulation of this type of instability is shown in Fig. 5. The numerical methods used are described by Davies and Carpenter.[37] The group velocity is zero and appears to have the attributes of an absolute instability. Davies and Carpenter[38] have shown that the transitional mode replaces divergence for laminar plane channel flow. It is known[39] that this also happens for the flat-plate boundary layer. In practice, an experimentalist would find it difficult to distinguish between divergence and the transitional mode as they are both absolute instabilities.

III. Multiple-Panel Compliant Walls

There is now a body of work based on theory and numerical simulations that suggests that T/S waves can be completely suppressed to indefinitely high Reynolds numbers by the use of mutiple-panel compliant walls. A series of compliant panels could be used, each with its properties tailored to suit the local flow environment. Carpenter and Morris[24,40] developed a methodology for the optimization of an infinitely long, plate-spring-type, compliant wall to achieve the maximum-possible transition delay. The essential concept underlying the optimization procedure is to use estimates for the onset speeds of traveling-wave flutter and divergence in order to restrict the choice of wall properties to those corresponding to marginal stability at the design flow speed with respect to those two flow-induced instabilities. Equations (1) and (2) were used for this purpose. The performance of a particular compliant-wall design for laminar-flow control is quantified by following Gyorgyfalvy[4] and introducing a transition delay factor (TDF):

$$\mathrm{TDF} = \frac{[(Re_{xt})]_{\text{compliant wall}}}{[(Re_{xt})]_{\text{rigid wall}}} = \left\{ \frac{[(Re_{\delta^*})_{e^n}]_{\text{cw}}}{[(Re_{\delta^*})_{e^n}]_{\text{rw}}} \right\}^2 \qquad (4)$$

The form of the TDF follows from the variation of δ^* with x for a laminar flat-plate boundary layer. The wall properties corresponding to the greatest TDF are then found from among this restricted set by using e^n-type calculations like those illustrated in Fig. 6. However, as discussed in Sec. II, it is now known that Eq. (2), which is based on a perturbed potential flow, is very conservative. We now know that in a laminar flow, keeping the wall properties fixed, the flow speed can be more than double the value given in Eq. (2) without divergence occurring. The results given in Fig. 6 were obtained by merely multiplying the right-hand side of the expression for U_d in Eq. (2) by 1.4. This modification allowed us to reduce the values of B and K (i.e., make the wall more compliant) while keeping $U_d = U_\infty$. This is a relatively minor relaxation compared with what we now know is possible. The method of estimating C_p reviewed in Sec. II was used to show that this new, more compliant, wall would still be marginally stable with respect to traveling-wave flutter, but that the flow speed would still be well below the onset speed for divergence. For the wall properties found with this slightly relaxed estimate for U_d, the methods of Carpenter and Morris[24,40] produce the amplification curves plotted in Fig. 6. Each curve corresponds to a T/S wave of fixed frequency. If one were to use these results as the basis of a transition-point prediction using the e^n method[41] with $n = 9.5$, a TDF of almost six would be obtained for this particular compliant wall. But the most noteworthy feature of Fig. 6 is the complete suppression of the T/S waves between approximately $Re_{\delta^*} = 2 \times 10^3$ and 2.6×10^3. In Fig. 21 of Ref. 24, with

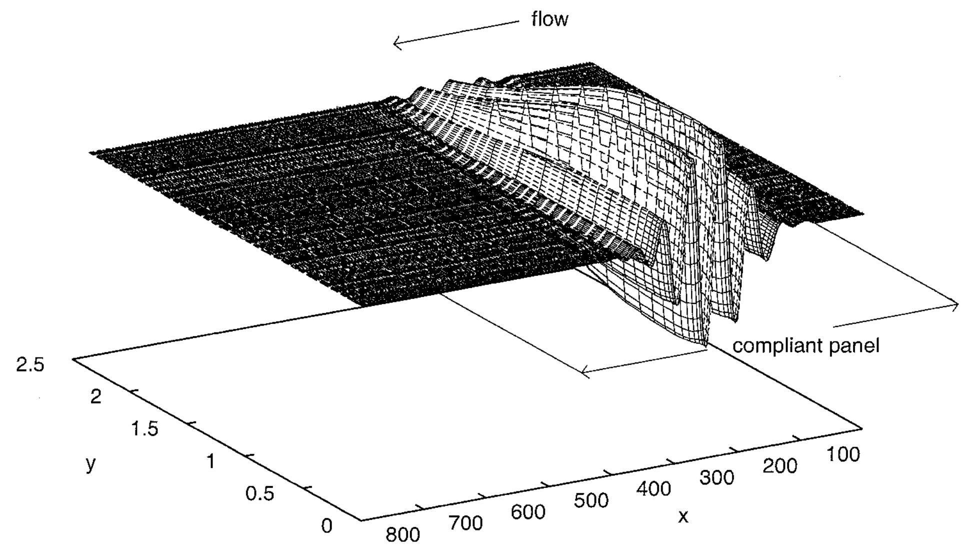

Fig. 5 Instantaneous spatial variation of the streamwise velocity perturbation for the absolutely unstable transitional mode over a laminar flat-plate boundary layer interacting with a plate-spring-type compliant panel. The results were obtained by numerical simulation.[39]

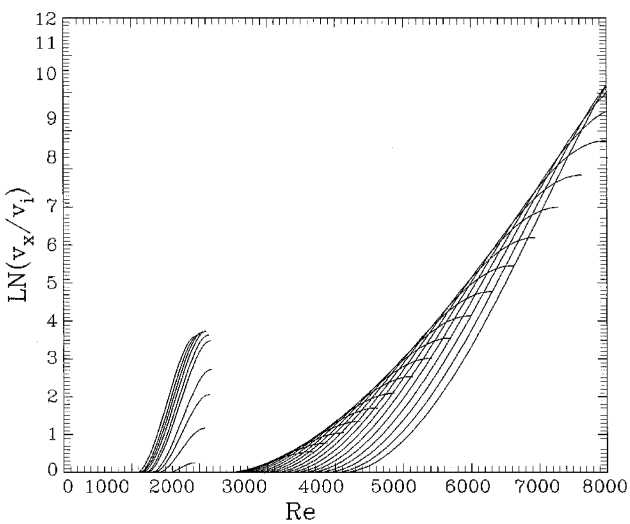

Fig. 6 Theoretical amplification curves for a plate-spring-type compliant wall with good laminar-flow-control capability. Each curve corresponds to a fixed frequency with the magnitude falling from left to right. v_x and v_i denote, respectively, the amplitudes of the velocity perturbations at the current streamwise station and the point on the lower branch of the neutral curve corresponding to the fixed value of frequency (based on Fig. 1 of Ref. 37).

optimization based on the slightly more conservative estimate of divergence speed given in Eq. (2), the same range of Re_{δ^*} corresponds to a minimum in amplification, but complete suppression was not observed.

The "window of T/S suppression" can be moved to higher values of Re_{δ^*} by changing the ratio of bending to spring stiffness of the compliant wall. In effect, the wall properties can be tailored to achieve complete suppression of the T/S waves. The best way to do this in practice would be to use a series of relatively short, locally tailored, compliant panels in the streamwise direction. (Carpenter[42] showed that even the use of only two compliant panels in series can bring considerable benefit compared with using a compliant wall with uniform properties.) Short compliant panels bring yet a further advantage because they are less vulnerable to aero- and hydroelastic intabilities.[43] Furthermore, should a locally severe, environmental disturbance be generated for some reason, the resulting turbulent flow and possible divergence would thereby be confined to a limited region. (The wedge of turbulent flow and divergence produced in the experimental study of Gad-el-Hak et al.[33]—see Sec. II—suggests that this is a feasible outcome.)

To be really effective at suppressing T/S waves, theory indicates that fairly short panels should be used. But the theory is based on the assumption that the compliant wall is infinitely long. An obvious question is: Do panels as short as a few, or even one, T/S wavelengths retain their capability to suppress T/S waves? This question has been investigated in detail through numerical simulation by Davies and Carpenter[37] in the case of plane channel flow. The same numerical techniques are used here to simulate the response of a short compliant panel when a T/S wave propagating along a flat-plate boundary layer is incident on the panel's leading edge. An example of one of these simulations is given in Fig. 7. It can be seen that the panel exhibits a complex response. Figure 7b is typical of a case where the frequency of the T/S wave is above the cutoff frequency of the compliant panel. If Fourier analysis is used to investigate the panel response in more detail, the spectrum for wall displacement shown in Fig. 7a can be obtained. This shows that the response is actually the result of a superposition of three separate eigenmodes of the boundary-layer/compliant-wall eigensystem. Vertical lines corresponding to the frequencies of the theoretically determined eigenmodes are drawn in Fig. 7a. The three superimposed eigenmodes are 1) the original T/S wave; 2) a damped, flow-induced surface wave that propagates upstream from the panel's trailing edge with a similar wavelength to the T/S wave; and 3) a lightly damped, near-neutral, much longer, flow-induced surface wave that propagates downstream from the panel's leading edge. Despite the complex response, the compliant panel in Fig. 7 does suppress the growth of the T/S wave, which emerges with much-reduced amplitude from the trailing edge.

In many respects Fig. 7 depicts an extreme case. No wall damping is included, and hinged-end conditions are assumed at the leading and trailing edges. The use of more realistic clamped end conditions considerably reduces the extreme behavior at the panel's leading edge. Including light wall damping reduces the complexity of the response. And, when the wall properties are selected so that the cutoff frequency is above the T/S frequency, the panel response is also much less complex. A typical example of a much milder response is given in Fig. 8. In this case clamped end conditions were assumed. Furthermore the panel is only about two T/S wavelengths long. Nevertheless the T/S waves are damped as they propagate over this short panel. The main outcome from our recent study is that the favorable conclusions drawn from our simulations of plane channel flow[37] concerning the use of short compliant panels can be carried over to boundary-layer flows. It appears that compliant panels as short as one T/S wavelength remain effective at suppressing T/S waves.

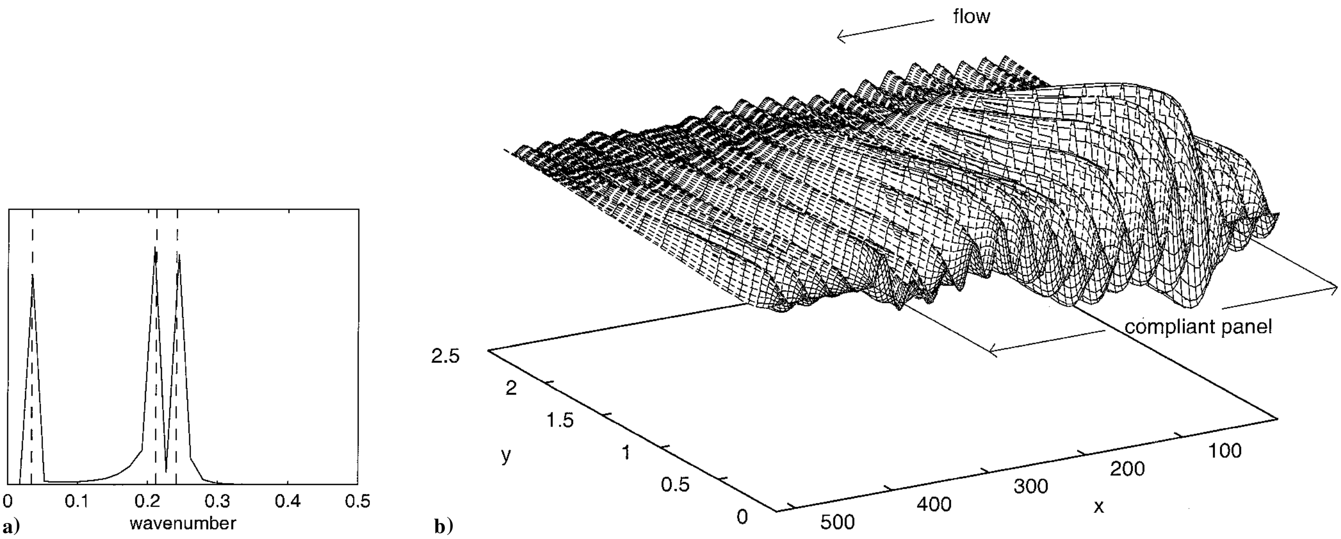

Fig. 7 Numerical simulation of a T/S wave propagating in a laminar flat-plate boundary layer over a finite compliant panel of plate-spring type: a) Power spectrum obtained by analyzing the instantaneous profile of wall displacement; the vertical broken lines correspond to eigenmodes of the coupled Orr–Sommerfeld/compliant-wall eigenproblem. b) Instantaneous spatial variation of the streamwise velocity perturbation plotted along and normal to the wall.

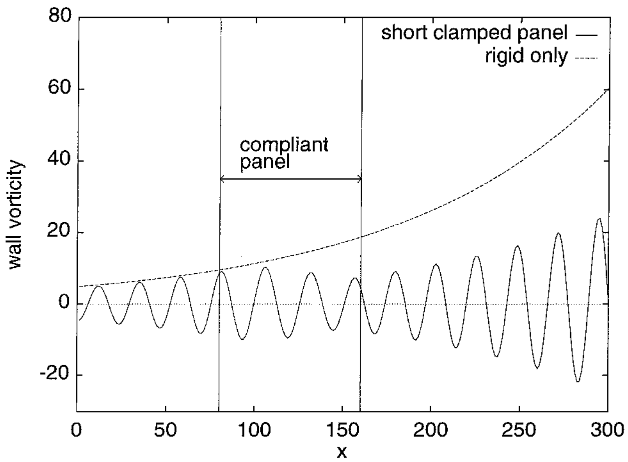

Fig. 8 Similar to Fig. 7 except that in this case the perturbation vorticity at the wall is plotted and the compliant panel is much shorter (its leading and trailing edges are indicated by the vertical lines) with clamped end conditions being used.

IV. Feasibility of Aeronautical Applications

A. Compliant Walls

In principle there is no reason why the theoretical methods just discussed should not be used to design compliant walls for laminar flow control in air. In water flow the properties required for near-optimum compliant walls have been demonstrated to be feasible for manufacture and are probably reasonably practicable for many marine applications. We will now assess the feasibility of using compliant walls for laminar flow control in aeronautical applications.

The most obvious difference between air and water is that the latter is 800 times more dense. Another significant difference is that the kinematic viscosity of air is 15 times greater than that of water. When the fluid density is greatly different from the density of the wall material, there are two main consequences that affect the physics of the wall/flow interaction. First, the interfacial condition equating the normal surface tractions in the fluid and solid requires an additional term to account for the body-force pertubation arising when the compliant surface is perturbed—see Eq. (4) of Ref. 8. In practice, this effect is usually unimportant. More significant is the mismatch between the wall and fluid inertias, which could come about when the densities are greatly different. For wall compliance to have a significant effect on T/S waves, the wall and fluid inertias must be of the same order of magnitude. This is well illustrated by the theoretical results carried out in Ref. 10 for comparison with the experimental study of Lee et al.[44] It was found that a single-layer, homogeneous, visco-elastic wall with a density of 1000 kg/m^3 was indistinguishable from a rigid wall in its effect on T/S waves. It was necessary to reduce the wall density to close to that of air, i.e., 1.2 kg/m^3, in order to see a significant stabilizing effect on the T/S waves. As mentioned in Sec. I, when compliant walls are optimized for laminar flow control their free-wave speed turns out to be about $0.7U_\infty$. For single-layer, visco-elastic walls the free-surface-wave speed is given approximately by

$$c_s \simeq 0.7\sqrt{G_s/\rho_s} \tag{5}$$

where G_s and ρ_s are the shear modulus and density, respectively, of the wall material. From Eq. (5) it is plain that the free-wave speed is strongly dependent on the material density.

Lee et al.[44] have carried out Gaster-type experimental studies on very soft, single-layer, compliant walls in a wind tunnel. They have apparently observed a stabilizing effect on the T/S waves. A fairly detailed comparison between the predictions of hydrodynamic stability theory and their experimental results was carried out in Ref. 10. Essentially the same theory was used, suitably modified to allow for the density differences between the wall and air, as previously corroborated by comparison with Gaster's experimental data.[22,23] According to the theory, the compliant walls of Lee et al. were effectively equivalent to a rigid wall as regards their effect on T/S waves. This does not mean that their results were not genuine, although there were some shortcomings from the point of view of making comparison with theory which were discussed in Ref. 10. What can be concluded, though, is that the observed stabilization is not a result of the physical mechanism found in Gaster's experiment and studied extensively by us and others.

Dixon et al.[11] have shown how the properties of single- and double-layer compliant walls can be optimized for obtaining the greatest possible transition delay. We will now see what properties their methods predict for application in air. For single-layer walls Dixon et al. showed that the Gyorgyfalvy transition-delay factor could not exceed about 2.5. As just mentioned, this could be achieved when $c_s \simeq 0.7U_\infty$. Thus, using Eq. (5) the required shear modulus is given by

$$G_s \simeq \rho_s U_\infty^2 \tag{6}$$

Table 1 Optimal compliant wall properties for use in airflow

Wall type	U_∞, m/s	ρ_p, kg/m^3	b, μm	E_p, GPa	ρ_s, kg/m^3	E_s, Pa
Single layer	25	——	——	——	1.2	3,000
Single layer	100	——	——	——	1.2	12,000
Single layer	200	——	——	——	1.2	48,000
Double layer	25	1,000	4	62,500	1.2	900
Double layer	25	5,000	0.8	8×10^6	1.2	900
Double layer	100	1,000	1	10^6	1.2	14,400
Double layer	100	5,000	0.2	125×10^6	1.2	14,400

To match the wall and fluid inertias, it would be necessary to use a material with density close to that of air. Assuming that a suitable such material exists, e.g., a type of aerogel, we see from Table 1 what values of elastic modulus ($E_s \simeq 3G_s$ for elastomers) are implied by Eq. (6). It is possible to obtain such elastic moduli for elastomeric materials such as silicone rubber, but for such materials the density is around 1000 kg/m^3. We do not know whether very light aerogel-like materials with these values of elastic modulus could be manufactured. What is certain is that the lack of robustness of walls made from such materials would make conventional aeronautical applications impractical.

Two-layer walls with an upper plate-like layer have a much better potential for laminar-flow control. Based on the results of Dixon et al.,[11] the optimal properties for the upper plate-like layer of such walls for transition delay in airflow are given approximately by

$$b \simeq 0.1/\rho_p U_\infty, \qquad E_p \simeq 100 U_\infty^2 \rho_p^3 \tag{7}$$

provided SI units are used throughout.

The estimates in Eq. (7) are used in Table 1 to give wall properties for plate material densities of 1000 kg/m^3 (typical of elastomers) and 5000 kg/m^3 (typical of metals). It can be seen that even at a low airspeed of 25 m/s the required elastic moduli for the plate are well beyond what is possible for existing materials. The gap between what is required and what is possible widens still further as the airspeed rises. Accordingly we conclude that it is not possible to manufacture a two-layer, compliant wall with a plate-like outer layer for aeronautical applications.

Before giving up completely, we should consider the possibility of using a tensioned membrane as the stiff outer layer instead of a plate. This, in fact, would follow directly in the footsteps of Gyorgyfalvy.[4] For this purpose we turn to the optimization procedures set out by Carpenter and Morris.[40,24] With this approach one is left with only two free wall parameters, after selecting the wall properties to meet the requirement of marginal stability with respect to traveling-wave flutter and divergence. One parameter is the nondimensional critical wave number for divergence $\bar{\alpha}_d$ [see Eq. (2)]; for a tensioned membrane over a spring foundation[21]

$$\alpha_d = \sqrt{K/T}, \qquad U_d = \left(4KT/\rho_f^2\right)^{\frac{1}{4}} \tag{8}$$

where T is the tension per unit width applied to the outer layer. The other free parameter is the dimensionless wall-damping coefficient. Thus, in the absence of wall damping, only $\bar{\alpha}_d$ is available to vary for optimization with respect to the stabilization of T/S waves. Figure 9, taken from Ref. 45, plots the dimensionless growth rate of the most rapidly growing T/S wave at a given value of Re_{δ^*} vs $\bar{\alpha}_d/\bar{\alpha}_{m0}$ (where $\bar{\alpha}_{m0}$ is the nondimensional wavenumber of the T/S wave) for three compliant walls. It is striking that in each case the optimum with respect to T/S growth rate corresponds to the same value of $\bar{\alpha}_d/\bar{\alpha}_{m0}$ and also to the same much-reduced growth rate. Also noteworthy is that the optimum is much shallower for the tensioned-membrane compliant wall, implying significant T/S wave stabilization for a much wider range of off-design conditions. Table 2 is also taken from Ref. 45; it gives the range of mechanical properties for a compliant wall comprising a tensioned membrane surmounting a spring foundation with a good laminar-flow capability at an airspeed of 25 m/s. Four cases are considered: 1) an elastomeric membrane with mass chosen so that $U_c = U_\infty$ using Eq. (1) for U_c (denoted

Table 2 Near-optimal wall properties for a compliant wall comprising a tensioned membrane over a spring foundation for airflow with $Re_\delta^* = 2240$

Membrane material	Relative membrane Inertia	U_∞, m/s	b, μm	T, N/m	K, kN/m^3
Elastomer	1	25	4–36	2.5–23	60–5
Elastomer	3	25	11–10	2.5–23	60–5
Titanium	1	25	0.8–8	2–23	60–5
Titanium	3	25	2.4–24	2–23	60–5

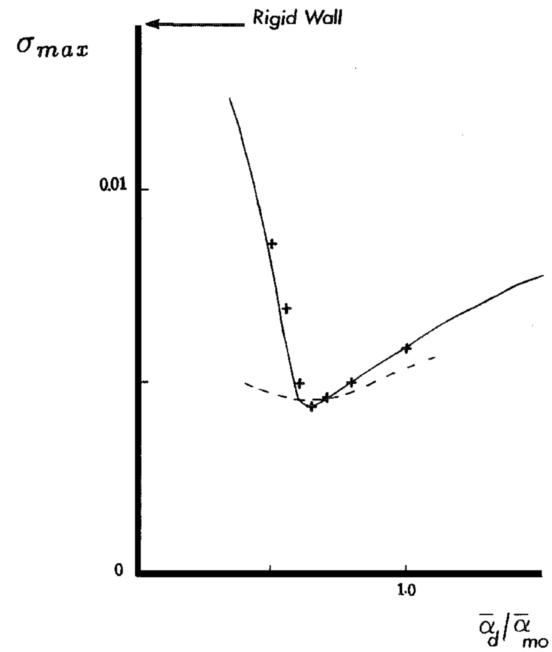

Fig. 9 Variation of T/S maximum growth rate at a fixed $Re_{\delta^*} = 2.24 \times 10^3$ with nondimensional divergence wave number for various optimal compliant walls: ——, plate/spring type ($T = 0$); – – –, tensioned membrane over spring foundation ($B = 0$); and +, plate-spring compliant wall with an infinitely deep inviscid fluid substrate (based on Fig. 2 of Ref. 45).

by one under relative membrane inertia in Table 2); 2) a similar membrane with mass three times this value (denoted by three under relative membrane inertia in Table 2), on the assumption that wall damping could be used to control the traveling-wave flutter; and 3) and 4) a titanium membrane with the two masses chosen as in the former two cases. To obtain the wall properties corresponding to other airspeeds, we note that

$$b \propto 1/U_\infty, \qquad T \propto U_\infty, \qquad K \propto U_\infty^3 \tag{9}$$

From these results it appears that it would just about be possible to manufacture a compliant wall with a membrane made from an elastomeric material for airspeeds up to about 50 m/s. The use of a metallic membrane seems to be just on the edge of what is practically possible. Arguably aluminum would be preferable to titanium; the range of thicknesses in this case would be 4–40 μm. Both elastomeric and metallic membranes would be extremely delicate, and this problem worsens with a rise in airspeed. Accordingly, we conclude that this lack of robustness makes it also infeasible to use membrane-type compliant walls for laminar flow control in aeronautical applications.

B. Other Flexible and Interactive Walls

Direct numerical simulations carried out by Metcalfe et al.[46] show that compliant walls, with properties close to optimal for the stabilization of T/S waves, continue to have a strong stabilizing effect in the nonlinear regime of transition. This suggests that such compliant walls would also act to reduce turbulence levels and skin friction in fully turbulent boundary layers. However, to the best of our knowledge, this has never been confirmed experimentally. What does appear to have been observed in a number of experimental investigations is that relatively stiff flexible walls (of the order of a 100 times stiffer than the Kramer or Gaster surfaces) with free surface wave speeds, which greatly exceed $0.7U_\infty$, do have a significant effect on fully turbulent boundary layers, producing reductions in turbulence intensity and/or drag of up to 20% or more.[47–51] The theory outlined in the preceding sections would indicate that such walls are indistinguishable from rigid walls in their effect on hydrodynamic stability. Plainly, then, if these observed drag reductions are a genuine effect of wall flexibility, some other physical mechanism is responsible, and it seems that it may also be effective in airflow.[49] The physical mechanism involved has not really been identified, although Semenov[47] has developed a semi-empirical theory based on Sternberg's[52] approach to the viscous sublayer. Very recently a group at Cornell University have developed a promising approach for studying this mechanism.[53,54]

Finally, recognizing that the problem with the use of compliant walls in air is the mismatch between the inertias of the solid and fluid, Carpenter and Porter[55,56] have proposed using a passive porous wall for laminar-flow control. Their passive porous walls take the form of a laser-drilled, stainless-steel membrane stretched over a plenum chamber. Typically the membrane has porosities in the range 6–12%. The basic concept is that as a T/S wave propagates along the boundary layer over the passive porous surface it will drive minute quantities of air in and out of the wall. This leads to a very similar effect at the wall as for a compliant surface. The slight outflow and inflow is equivalent, for small displacements, to the upward and downward velocity of the compliant surface. For the passive porous wall it is the air rather than solid wall that moves in and out, thereby overcoming the problem of the mismatch in inertias. Despite the superficial analogy between compliant and passive porous walls, however, Carpenter and Porter[55,56] showed that the phase differences between the pressure and normal velocity perturbation in the two cases were opposite in sign, implying radically different dynamics. Nevertheless, it appears that, in theory, appropriately designed and practically realizable passive porous surfaces could suppress the growth of T/S waves. An optimum porosity of about 12% was predicted by the theory.

An experimental study[57] was carried out at Queen Mary and Westfield College, London, in collaboration with Professor M. Gaster and British Aerospace in order to confirm the theory. This found a small amount of stabilization for porosities of the order 6%. For higher levels of porosity, a powerful feedback mechanism came into play whereby pressure fluctuations were generated in phase with inflow and outflow from the plenum chamber. This produced large-amplitude and highly coherent, three-dimensional flow structures, which appeared to be similar to K-type lambda vortices. This phenomenon was very robust and could not be easily disrupted. Its occurrence prevented the theoretically predicted suppression of T/S waves from being observed experimentally. Plainly, some means of eliminating the feedback phenomenon will have to be devised in order for these passive porous walls to provide a viable laminar flow-control technique.

V. Conclusions

The main conclusions to be drawn from our review and assessment of recent progress toward the use of compliant walls for laminar-flow control are as follows:

1) Good agreement has been found between the theory and the experimental study of Gaster for the development of Tollmien–Schlichting waves in water flow over a compliant panel inset into a flat plate. Furthermore there is close agreement between theory and experiment regarding the onset of the flow-induced surface instabilities. Thus there is independent experimental confirmation of theory.

2) The theory indicates that traveling-wave flutter is the crucial instability as regards the application of compliant walls for laminar-flow control. It was the route to transition for two of the three compliant panels in Gaster's experiments. The main role of wall damping is to control this instability so that a more compliant surface can be used.

3) New results from numerical simulations are presented for the various flow-induced instabilities. In particular, these show how the amplitude of pressure fluctuations is greatly reduced by a laminar boundary layer, leading to revised estimates for the onset of divergence instability, which is now found to be much less powerful in laminar boundary layers than previously thought.

4) Theory and numerical simulations suggest that it may well be possible to suppress Tollmien–Schlichting waves completely at any Reynolds number no matter how high. New results are presented showing that very short compliant panels can still suppress the growth of Tollmien–Schlichting waves.

5) The feasibility of using compliant walls for laminar-flow control in aeronautical applications has been assessed. It is concluded that such delicate walls are required in order to match the inertias of the wall and air so that the lack of robustness makes the use of compliant walls in air impractical. The use of other types of flexible and interactive walls is also assessed.

There are three outstanding issues that remain to be addressed in order to make the use of complaint walls practical for laminar flow control in marine applications. First, elastomeric materials which have the appropriate mechanical properties and are suitable for long-term use, need to be developed. Experimental studies of multiple-panel complaint walls are desirable to test the concept. Controlled tests of compliant walls in a real marine environment and at practical design conditions are also desirable. Finally, the outstanding theoretical questions concern receptivity. How does wall compliance affect receptivity to freestream turbulence and particulate matter? This is the subject of our current research.

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